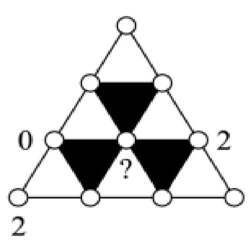
UT Arlington Mid-Cities Math Circle $(MC)^2$
Puzzles and More
February 16, 2022

Warm-up problems

Problem 1. In the 5×5 square shown below the sum of the numbers in each row and in each column is the same. There is a number in every cell, but some of the numbers are not shown. What is the number in the cell marked with a question mark?

	16		22	
20		21		2
	25		1	
24		5		6
	4		?	

Problem 2. Each of the ten points in the diagram is labelled with one of the numbers 0, 1 or 2. It is known that the sum of the numbers in the corner points of each white triangle is divisible by 3, while the sum of the numbers in the corner points of each black triangle is not divisible by 3. Three of the points are already labeled as shown in the diagram. With which numbers can the inner point be labeled?



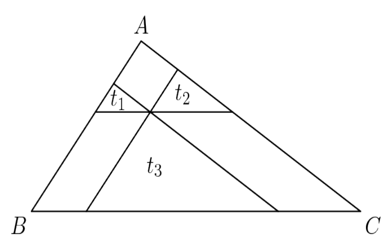
Problem 3. Tom has a collection of 13 snakes, 4 of which are purple and 5 of which are happy. He observes that all of his happy snakes can add, none of his purple snakes can subtract, and all of his snakes that can't subtract also can't add. Which of these conclusions can be drawn about Tom's snakes?

(A) Purple snakes can add.
(B) Purple snakes are happy.
(C) Snakes that can add are purple.
(D) Happy snakes are not purple.
(E) Happy snakes can't subtract.

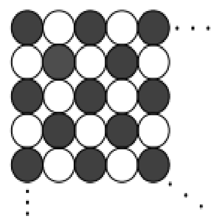
Harder problems

Problem 4. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of $90°$ counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

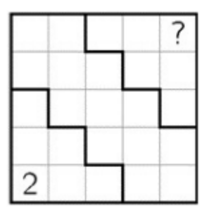
Problem 5. A point P is chosen in the interior of $\triangle ABC$ such that when lines are drawn through P parallel to the sides of $\triangle ABC$, the resulting smaller triangles t_1, t_2, and t_3 in the figure, have areas 4, 9, and 49, respectively. Find the area of $\triangle ABC$.



Problem 6. Julia has 2017 round discs available: 1009 black ones and 1008 white ones. Using them, she wants to lay the biggest square pattern (as shown) possible and starts by using a black disc in the left upper corner. Subsequently she lays the discs in such a way that the colors alternate in each row and column. How many discs are left over when she has laid the biggest square possible?



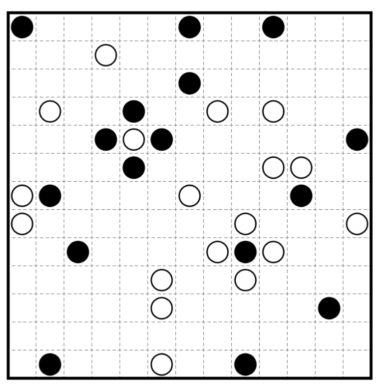
Problem 7. Numbers are to be placed into the square grid shown below, so that each of the numbers 1, 2, 3, 4 and 5 appears exactly once in each row and in each column. Furthermore, the sum of all numbers in the three black-bordered sections should always be the same. Which number has to be written into the top right cell?



Problem 8. Recall the rules of the classical Sudoku. One needs to place a digit from 1 to 9 in each empty cell in the grid such that each row, column, and marked 3×3 box contains each digit exactly once. Find the exact number of solutions to the sudoku below.

6	8			3			9	7
9	7			6			3	
		5	8			1		
		9	7			2		
1	6			5	2			4
				4	6			1
		3	6					9
8	1							3
7				2	3	8	1	

Problem 9. (Compliments of Luke Robitaille) Masyu is played on a rectangular grid of squares, some of which contain circles; each circle is either "white" (empty) or "black" (filled). The goal is to draw a single continuous non-intersecting loop that properly passes through all circled cells. The loop must "enter" each cell it passes through from the center of one of its four sides and "exit" from a different side; all turns are therefore 90 degrees. Solve the following Masyu puzzle.



Problem 10. Let Γ be a circle with centre I, and $ABCD$ a convex quadrilateral such that each of the segments AB, BC, CD and DA is tangent to Γ. Let Ω be the circumcircle of the triangle AIC. The extension of BA beyond A meets Ω at X, and the extension of BC beyond C meets Ω at Z. The extensions of AD and CD beyond D meet Ω at Y and T, respectively. Prove that

$$AD + DT + TX + XA = CD + DY + YZ + ZC$$