UT Arlington Mid-Cities Math Circle $(MC)^2$
Selected 2009 AMC10 Problems

Problem 1. (12/10A) In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5, DA = 9$, and BD is an integer. What is BD?

Problem 2. (17/10A) Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that EF is perpendicular to DB, and A and C lie on DE and DF, respectively. What is EF?

Problem 3. (23/10A) Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

Problem 4. (16/10B) Points A and C lie on a circle centered at O, each of $\overline{BA}$ and $\overline{BC}$ are tangent to the circle, and $\triangle ABC$ is equilateral. The circle intersects $\overline{BO}$ at D. What is $\frac{BD}{BO}$?

Problem 5. (18/10B) Rectangle $ABCD$ has $AB = 8$ and $BC = 6$. Point M is the midpoint of diagonal $\overline{AC}$, and E is on AB with $\overline{ME} \perp \overline{AC}$. What is the area of $\triangle AME$?

Problem 6. (20/10B) Triangle ABC has a right angle at $B, AB = 1$, and $BC = 2$. The bisector of $\angle BAC$ meets $\overline{BC}$ at D. What is BD?

Problem 7. (15/10A) The figures $F_1, F_2, F_3, ..$ form a sequence of figures. F_1 is a point, F_2 is a square, and for $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?

Problem 8. (25/10A) For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

Problem 9. (11/10B) How many 7-digit palindromes (numbers that read the same backward as forward) can be formed using the digits 2, 2, 3, 3, 5, 5, 5?

Problem 10. (24/10A) What is the remainder when $3^0+3^1+3^2+\cdots+3^{2009}$ is divided by 8?

Problem 11. (25/10B) Each face of a cube is given a single narrow stripe painted from the center of one edge to the center of the opposite edge. The choice of the edge pairing is made at random and independently for each face. What is the probability that there is a continuous stripe encircling the cube?

Problem 12. (23/10B) Rachel and Robert run on a circular track. Rachel runs counterclockwise and completes a lap every 90 seconds, and Robert runs clockwise and completes a lap every 80 seconds. Both start from the same line at the same time. At some random time between 10 minutes and 11 minutes after they begin to run, a photographer standing inside the track takes a picture that shows one-fourth of the track, centered on the starting line. What is the probability that both Rachel and Robert are in the picture?

More Challenging Problems

Problem 13: 2009 AIME II, Problem 5. Equilateral triangle T is inscribed in circle A, which has radius 10. Circle B with radius 3 is internally tangent to circle A at one vertex of T. Circles C and D, both with radius 2, are internally tangent to circle A at the other two vertices of T. Circles B, C, and D are all externally tangent to circle E, which has radius $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Problem 14: 2009 AIME II, Problem 12. From the set of integers $\{1, 2, 3, \ldots, 2009\}$, choose k pairs $\{a_i, b_i\}$ with $a_i < b_i$ so that no two pairs have a common element. Suppose that all the sums $a_i + b_i$ are distinct and less than or equal to 2009. Find the maximum possible value of k.