UT Arlington Mid-Cities Math Circle $(MC)^2$
Number Theory Problems II

Problem 9. How many primes, written in base 10, are such that their digits are alternating 1s and 0s, beginning and ending with 1?

Problem 10. Prove that for some $k > 0$ the k-th term F_k of the Fibonacci sequence is a multiple of 2011^{2011}.

Problem 11. Suppose that the positive integers x, y satisfy $2x^2+x = 3y^2+y$. Show that $x - y$, $2x + 2y + 1$, $3x + 3y + 1$ are all perfect squares.

Problem 12. (Putnam 1975, A1.) For positive integers n define $d(n) = n - m^2$, where m is the greatest integer with $m^2 \leq n$. Given a positive integer b_0, define a sequence b_i by taking $b_{k+1} = b_k + d(b_k)$. For what b_0 do we have b_i constant for sufficiently large i?

Problem 13. (USAMO 1979) Find all non-negative integral solutions $(n_1, n_2, ..., n_{14})$ to $n_1^4 + n_2^4 + ... + n_{14}^4 = 1599$.

Problem 14. Prove that there exist infinitely many integers n for which $2^n + 1$ is divisible by n.

Problem 15. Prove that the equation $x^2 = y^3 + 7$ has no integer solutions.