The pigeonhole principle states that if \( n \) items are put into \( m \) pigeonholes with \( n > m \), then at least one pigeonhole must contain more than one item (items=pigeons).

**Warm-up 1.** If each point of the plane is colored red or blue then there are two points of the same color at distance 1 from each other.

**Warm-up 2.** Among 13 persons, there are two born in the same month.

**Problem 1.** If there are \( n \) number of people who can shake hands with one another (where \( n > 1 \)), then there is always a pair of people who will shake hands with the same number of people.

**Problem 2.** Prove that however one selects 55 integers \( 1 \leq x_1 < x_2 < ... < x_{55} \leq 100 \), there will be some two that differ by 9, some two that differ by 10, a pair that differ by 12, and a pair that differ by 13. Surprisingly, there need not be a pair of numbers that differ by 11.

**Problem 3.** Prove that any \((n+1)\)-element subset of \( \{1, 2, ..., 2n\} \) contains two integers that are relatively prime.

**Problem 4.** (Putnam Exam 1978) Let \( A \) be any set of 20 distinct integers chosen from the arithmetic progression \( \{1, 4, 7, ..., 100\} \). Prove that there must be two distinct integers in \( A \) whose sum if 104.

**Problem 5.** During a month with 30 days a baseball team plays at least a game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

**Problem 6.** Prove that among any seven real numbers \( y_1, ..., y_7 \), there are two, \( y_i \) and \( y_j \), such that

\[
0 \leq \frac{y_i - y_j}{1 + y_i y_j} \leq \frac{1}{\sqrt{3}}.
\]

**Problem 7.** Prove that among five different integers there are always three with sum divisible by 3.