UT Arlington Mid-Cities Math Circle $(MC)^2$

Combinatorics II

Problem 9. Prove the identity

$$\binom{n}{0} + \binom{n}{1} + ... + \binom{n}{n-1} + \binom{n}{n} = 2^n$$

using the combinatorial defintion of $\binom{n}{k}$.

Problem 10. A person has 10 friends. Over several days he invites some of them to a dinner party in such a way that he never invites exactly the same group of people. How many days can he keep this up, assuming that one of the possibilities is to ask nobody to dinner?

Problem 11. How many ways are there to write n as a sum $n = a_1 + a_2 + \ + a_k$ such that each a_i is an odd positive integer number?

Problem 12. A lord had 10 guests for dinner. Each guest gave his hat to a valet. The lord is a practical joker. After the dinner he asked his valet to give the hats back so that none of the guests gets his own hat. How many ways are there to do it?

Problem 13. There are 10 lines on a plane that cut the plane into parts. Assume that every two lines intersect and there is no triple intersections. Find the number of parts. How many of these parts are bounded?

Problem 13. Given n planes in a space, any two planes meet in a line, any three planes meet at a point, any four planes do not have common points. Find the number of parts in which the planes divide the space. Find the number of bounded parts.