UT Arlington Mid-Cities Math Circle $(MC)^2$
Number Theory Problems

"Mathematics is the queen of the sciences and number theory is the queen of mathematics" –Gauss

Problem 1. Find the largest prime factor of:

$$3(3(3(3(3(3(3(3(3+1)+1)+1)+1)+1)+1)+1)+1)+1)+1$$

Problem 2. Compute: $199919981997^2 - 2 \cdot 199919981994^2 + 199919981991^2$.

Problem 3. The Fibonacci sequence is defined as follows: $F_0 = 0, F_1 = 1$, and if $n > 1$, $F_n = F_{n-1} + F_{n-2}$. The first few terms in the sequence are:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...$$

For which n is F_n divisible by 2? by 3? by 5? by 7? by 11?

Problem 4. Show that $2222^{5555} + 5555^{2222}$ is divisible by 7.

Problem 5. Find all natural numbers x, y, and z satisfying the following equation

$$x^x + y^y = z^z$$

(Reminder: A number n is natural if it is a positive integer.)

Problem 6. Find all integers x and y for which $x^2 - 3y^2 = 17$.

Problem 7. Prove that no three integers x, y, z satisfy

$$x^3 + y^3 + z^3 = 500$$