

UT Arlington Mid-Cities Math Circle (MC)²
Number Theory Problems 2

“Mathematics is the queen of the sciences and number theory is the queen of mathematics” –Gauss

Problem 8. Prove that

$$2^{70} + 3^{70}$$

is divisible by 13.

Problem 9. Prove that

$$2^{2^{6k+2}} + 3$$

is divisible by 19 for $k = 0, 1, 2, \dots$

Problem 10. Prove that for any integer n the number

$$5^{5^{n+1}} + 5^{5^n} + 1$$

is not a prime number.

Problem 11. Prove that for all positive integers n the equation

$$x^2 + y^2 + z^2 = 59^n$$

has integer solutions.

Problem 12. Define the sequence $a_1, a_2, \dots, a_n, \dots$ by $a_1 = 1$ and

$$a_{n+1} = 2a_n + \sqrt{3a_n^2 - 2}$$

for all $n \geq 1$. Prove that a_n is an integer for all n .

Problem 13. Prove that there exist infinitely many integers n for which

$$2^n + 1$$

is divisible by n .